Summation of Series

and

Permutation and Combination

using

Vedic Method

By Panna Lal Patodia

Summation of Series:

Below, we give Vedic method to find n^{th} term and summation of n terms. The method is based on the Vedic sutra "**Sisyate Sesasamjnah**" means "**Remainders remains constant**". For explaining the method, let us give some examples:

Example 1: Find n^{th} term and sum of first n terms of the series: 2,5,8,11

The given series is in Arithmetic Progression (AP) and can be computed easily using the following formulas:

n^{th} term = a + (n-1)d = 2 + (n-1)3 = 2 + 3n – 3 = 3n-1

Sum = n/2 [2a + (n-1)d] = n/2 [2*2 + (n-1)3] = n*(3n+1)/2

In Vedic method, we compute the successive difference till we arrive constant term and then apply the following formulas:

n^{th} term = First Diff + (n-1)*Second Diff + (n-1)*(n-2)/2*Third Diff
+ (n-1)*(n-2)*(n-3)/6*Fourth Diff + ...

Summation = n*First Diff + n*(n-1)/2*Second Diff
+ n*(n-1)*(n-2)/6*Third Diff
+ n*(n-1)*(n-2)*(n-3)/24*Fourth Diff + ...

Let us take the above series and apply the Vedic Method:

```
2     5     8     11
   3     3     3
```

Very Important: Note that the difference 3 is constant in second line. As per Vedic Method, we need to compute the difference till we reach constant term.

First Diff is the first value in first line. The Second Diff is the first value in second line. So, first Diff is 2 and second Diff is 3. Now, let us apply the vedic method:

n^{th} term = 2 + (n-1)*3 = 3*n – 1

Summation = n*2 + n*(n-1)*3/2 = n*(3n+1)/2

Example 2: Find n^{th} term and sum of first n terms of the series: 3,5,8,13,21,33,50

```
3     5     8     13     21     33     50
   2     3     5      8     12     17
      1     2     3      4      5
         1     1     1      1
```

From the above, it is apparent that

First Diff = 3
Second Diff = 2
Third Diff = 1
Fourth Diff = 1

Now, let us apply Vedic Method (Readers might note that this is not AP Series, so AP formulas are not applicable to it):

n^{th} term = 3 + (n-1)*2 + (n-1)(n-2)/2 + (n-1)(n-2)(n-3)/6

$= (n^3 – 3n^2 + 14n + 6)/6$

Summation = 3*n + 2*n(n-1)/2 + n(n-1)(n-2)/6 + n(n-1)(n-2)*(n-3)/24

$= n(n^3 – 2n^2 + 23n + 50)/24$

To arrive the same result using modern method, we need to complicated computation. **The formulas provided by Vedic Mathematics are easy to remember and have universal application.**

In next article, we shall cover how to compute n^{th} term and summation of series if the last line contains the values in Geometrical Progression instead of constants.

Permutation and Combination:

Now let take some examples of permutation and combination. We shall be solving these using Vedic methods, which are far simpler.

Example 1:
Find the sum of all the numbers formed by taking all the digits from 2,3,4 and 5.

Before we solve the above example using Vedic method, let us understand that we can apply the methods based on the different cases.

Case I - Conditions:

1. **Zero is not used. The numbers given should not have 0 as number.**
2. **No digit is repeated. Each digit is unique.**
3. **Summation to be computed taking all the digits.**

How to Solve?
Step 1: We compute the numbers can be formed with the given digits. Suppose, the number of digits are n, then numbers can be formed = n!

Step2: Unit Sum = n! * Sum of digits / Number of digits

Step 3: We write the unit sum n times vertically leaving one space and then compute the sum.

Let us show the working by solving Example 1.

As the above example satisfies all the conditions of Case I, we shall applying this.

Solution of Example 1:

Numbers can be formed with 4 digits (2,3,4 and 5) = 4! = 24

Unit Sum = 4! * (2+3+4+5)/4 = 84

The required sum (Write 84 into 4 lines):

```
84
 84
  84
   84
-----
93324
-----
```

So, the sum of all the numbers formed by taking all the digits from 2,3,4 and 5 is 93,324.

Let us take one more example:

Example 2:
Find the sum of all the numbers formed by taking all the digits from 2 to 9.

Solution:
Number of digits from 2 to 9 is 8. So, n = 8

Numbers can be formed = 8! = 40320

Unit Sum = 40320 * (2+3+4+5+6+7+8)/8 = 221760

Required Sum:

```
221760
 221760
  221760
   221760
    221760
     221760
      221760
       221760
-------------
2463999975360
-------------
```

Case II - Conditions:

1. **Zero is not used. The numbers given should not have 0 as number.**
2. **No digit is repeated. Each digit is unique.**
3. **Summation to be computed not taking all the digits.**

How to Solve?

Step 1: We compute the numbers can be formed with the given digits. Suppose, the number of digits are n, and the summation to be formed by taking m number (m < n), then numbers can be formed = n!/(n-m)!

Step2: Unit Sum = n! * Sum of digits / Number of digits

Step 3: We write the unit sum n times vertically leaving one space and then compute the sum.

Note: Step 2 and Step 3 are same.

Example 3:

Find the sum of all the 4 digit numbers formed by taking the digits from 1, 2, 5, 6, 8 and 9.

Solution:

Number of digits are 6. So, n = 6. Sum to be formed using 4 digits. So, m = 4

Numbers can be formed = 6!/(6-4)! = 6!/2! = 360

Unit Sum = 360 * (1+2+5+6+8+9)/6 = 1860

Required Sum:

```
1860
 1860
  1860
   1860
-------
2066460
-------
```

There are many more cases where Vedic Methods can be applied and the problem can be solved in a very simple way. We shall be discussing these in next article.

Summation of Series:

Below, we give Vedic method to find n^{th} term and summation of n terms. The method is based on the Vedic sutra "Sisyate Sesasamjnah" meaning, "Remainders remain constant". For explaining the method, let us give some examples:

Example 1: Find n^{th} term and sum of first n terms of the series: 2, 5, 8, 11

The given series is in Arithmetic Progression (AP) and can be computed easily using the following formulae:

$$n^{th} \text{ term} = a + (n-1)d = 2 + (n-1)3 = 2 + 3n - 3 = 3n - 1$$

$$\text{Sum} = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}\left[2 \times 2 + (n-1) \times 3\right] = \frac{n(3n+1)}{2}$$

In the Vedic method, we compute the successive difference till we arrive at the constant term and then apply the following formulas:

$$n^{th} \text{ term} = D_0 + (n-1)D_1 + \frac{(n-1)(n-2)}{2}D_2 + \frac{(n-1)(n-2)(n-3)}{6}D_3 + \cdots$$

where D_0 is the first term, D_1 the first difference, etc.

$$\text{Summation} = nD_0 + \frac{n(n-1)}{2}D_1 + \frac{n(n-1)(n-2)}{6}D_2 + \frac{n(n-1)(n-2)(n-3)}{24}D_3 + \cdots$$

Let us take the above series and apply the Vedic Method:

```
2     5     8     11
   3     3     3
```

Note that the difference 3 is constant in second line. As per Vedic Method, we need to compute the difference till we reach constant term.

In this example, D_0 is 2 and D_1 is 3. Now, let us apply the Vedic method:

$$n^{th} \text{ term} = 2 + (n-1) \times 3 = 3n - 1$$

$$\text{Summation} = n \times 2 + \frac{n(n-1)}{2} \times 3 = \frac{n(3n+1)}{2}$$

Example 2: Find n^{th} term and sum of first n terms of the series: 3, 5, 8, 13, 21, 33, 50

3		5		8		13		21		33		50
	2		3		5		8		12		17	
		1		2		3		4		5		
			1		1		1		1			

From the above, it is apparent that $D_0 = 3$, $D_1 = 2$, $D_2 = 1$ and $D_3 = 1$.

Now, let us apply Vedic Method (Readers might note that this is not AP Series, so AP formulas are not applicable to it):

$$n^{th}\text{ term} = 3 + (n-1)\times 2 + \frac{(n-1)(n-2)}{2} + \frac{(n-1)(n-2)(n-3)}{6}$$

$$= \frac{n^3 - 3n^2 + 14n + 6}{6}$$

$$\text{Summation} = 3n + \frac{2n(n-1)}{2} + \frac{n(n-1)(n-2)}{6} + \frac{n(n-1)(n-2)(n-3)}{24}$$

$$= \frac{n\left(n^3 - 2n^2 + 23n + 50\right)}{24}$$

To arrive at the same result using the conventional method, we need to do complicated computations. The formulas provided by Vedic Mathematics are easy to remember and have universal application.

Panna Patodia