IAVM

Sample Open IVMO
Time allowed - 1 hour

1. Subtract

$$\begin{array}{r} 95747 \\ -\ 65748 \\ \hline \end{array}$$

2. Draw a ring around the number in the list that is divisible by 9.

511765

763521

420673

485316

453688

3.

$$\begin{array}{r} 96 \\ \times 91 \\ \hline \end{array}$$

4.

$$\begin{array}{r} 107 \\ \times 112 \\ \hline \end{array}$$

5. 43×47

6.

$$\begin{array}{r} 106 \\ \times\ 93 \\ \hline \end{array}$$

7. 997^2

8. $23.6 \div 5$

9. 65^2

10. 27×72

11. Divide,

$$9\overline{)4261}$$

12. Divide,

$$11\overline{)47539}$$

13. Divide,

$$879\overline{)1010003}$$

14.

$$\begin{array}{r} 312 \\ \times\ 308 \\ \hline \end{array}$$

15. 273×516

16. 9.8×0.00093

17. How many pieces of wire, each 31 cm long can be cut from a roll of length 100 metres and what will be the remainder?

18. 16.8% of 25

19. Convert $\frac{116}{125}$ to decimal.

20. Convert the fraction, $\frac{7}{19}$, to decimal.

21. How many different digits are there when $\frac{17}{11}$ is converted to a decimal? (Draw a ring round the correct answer.)

A 2 B 3 C 4 D 5 E 6

22. Write the following fractions in order of size, starting with the smallest:

$$\frac{1}{113} \quad \frac{2}{225} \quad \frac{4}{447} \quad \frac{2}{227}$$

23. Express 85 as the difference of two square numbers that are integers, in two different ways.

24. Tara attempted to multiply 67×58 but wrote down one digit in the two numbers incorrectly and got the answer 3596. Find the sum of the two numbers she multiplied.

25. Marie goes to a shop where all the items have prices ending in 99 cents (for example, \$0.99, \$5.99, \$2.99). She spends \$33.89. How many items does she buy?

26. How many non-recurring decimal digits are there in the decimal equivalent of $\frac{101}{475}$?

27. Find the cube root of the exact cube, 238,328

28. 0.79^3

29. Exactly one of these equations is correct. Draw a circle round the correct one.

A $44^2 + 77^2 = 4477$

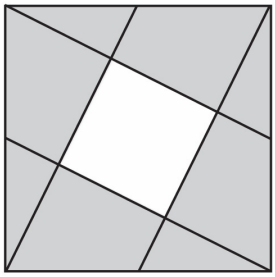
B $55^2 + 66^2 = 5566$

C $66^2 + 55^2 = 6655$

D $88^2 + 33^2 = 8833$

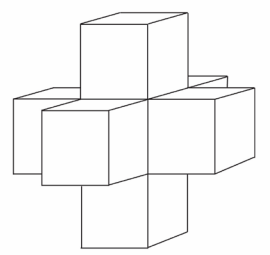
E $99^2 + 22^2 = 9922$

30. What fraction of the square is left unshaded?



31. The solid as shown is made by fixing cubes on each face of a central cube. The solid has a volume of $875\,cm^3$.

What is the surface area of the solid?



32. Two similar cones, A and B, have surface areas $900\,cm^2$ and $8100\,cm^2$, respectively.

If the volume of cone A is $1800\,cm^3$, what is the volume of cone B?

33. $99^3 + 3 \times 99^2 + 9 \times 33 + 1$

34. Is 232808 divisible by 49?

35. If $6x - y = 21$ and $6y - x = 14$, what is the value of $x - y$?

36. Expand and simplify,

$$(x^2 - 5x + 3)(3x^2 + 7x - 4)$$

37. Given that $(x - 2)$ is a factor of,

$$f(x) = 6x^3 - 19x^2 + 9x + 10$$

find the solutions to $f(x) = 0$.

38. $$4x^3 + 8x^2 + 9x + 10 \div (2x + 3)$$

39. Find the equation of the straight line with gradient, 3, and that passes through the point (4, 2).

40. Find the equation of the straight line perpendicular to the line with equation, $3x + 5y = 23$ and which passes through the point (1, 5).

41.
$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \cdots = \frac{\pi}{4}$$
$$\frac{1}{1\times 3} + \frac{1}{5\times 7} + \frac{1}{9\times 11} + \cdots = \frac{\pi}{x}$$

What is *x*?

42. *a* and *b* are real numbers such that,

$$a \div b = a \times b = a + b$$

Find the value of $a - b$.

43. Five numbers are arranged in order from least to greatest.

$$x \quad x^3 \quad x^4 \quad x^2 \quad x^0$$

Where does $-x^{-1}$ belong in the list above?

44. A circle has equation, $x^2 + y^2 = 9$. In how many places does the straight line, with equation $x + 2y = 7$, cross the circumference of the circle, twice, once or no times?

45. $\left(3^{23} - 3^{22}\right)\left(3^{24} - 3^{23}\right)\left(3^{25} - 3^{24}\right) = 2^m \times 3^n$

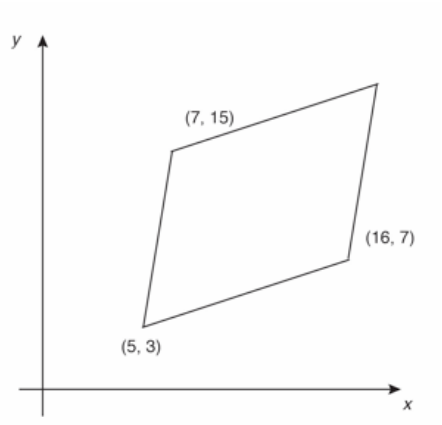
Find the value of $m + n$

46. If a, b, c and d are the roots of the equation,

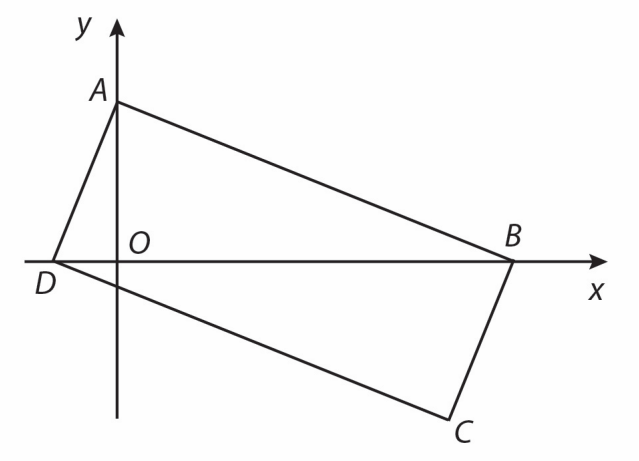
$$\left(\frac{13x - x^2}{x+1}\right)\left(x + \frac{13 - x}{x+1}\right) = 42$$

Find the value of $2020(a + b + c + d)$.

47. Find the area of the parallelogram given the coordinates of three vertices as shown.



48.



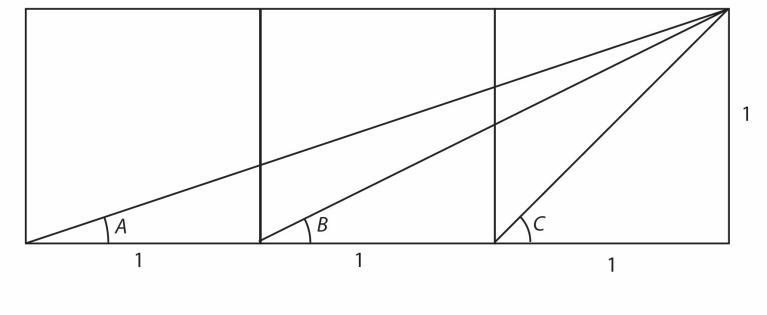
The figure shows a rectangle, *ABCD*. The equation of the line *AB* is $2x + 5y = 10$. The point *A* lies on the y-axis and points *B* and *D* lie on the x-axis.

Work out the area of the rectangle.

49. Evaluate,

$$\tan^{-1}\tfrac{1}{2} + \tan^{-1}\tfrac{1}{5} + \tan^{-1}\tfrac{1}{8}$$

50.



Find the sum of the three angles A, B and C.

Sample Open IVMO
Answers

1. Subtract

$$\begin{array}{r} 95747 \\ -\ 65748 \\ \hline \end{array}$$

29999

By inspection

2. Draw a ring around the number in the list that is divisible by 9.

511765

763521

420673

485316

453688

By elimination and retention

3.

$$\begin{array}{r} 96-04 \\ \times\ 91-09 \\ \hline 87\quad 36 \end{array}$$

All from 9 and the last from 10

4.

$$\begin{array}{r} 107+07 \\ \times\ 112+12 \\ \hline 119\quad 84 \end{array}$$

All from 9 and the last from 10

5. $43 \times 47 = 2021$

By one more than the one before

When the final digits add to 10

6.

$$\begin{array}{r} 106+06 \\ \times\ 93-07 \\ \hline 99\quad \overline{4}\,\overline{2} \\ \hline 98\quad 58 \end{array}$$

All from 9 and the last from 10

7. 997^2 994009

Whatever the deficiency, lessen it further, and set up the square

8. $23.6 \div 5$

4.72

Proportionately

9. 65^2

4225

By one more than the one before

10.

$$\begin{array}{r} 27 \\ \times\ 72 \\ \hline 19_5 4_1 4 \end{array}$$

Vertically and crosswise

11. Divide,

$9 \mid 4\ 2\ 6\ 1$

473/4

All from 9 and the last from 10

12. Divide,

$11 \mid 4\ 7\ 5\ 3\ 9$

4321/8

Transpose and apply

13. Divide,

879	101/003
121	12 1
	1 21
	484

114 /797

All from 9 and the last from 10

14.

$$\begin{array}{r} 312 \\ \times\ 308 \\ \hline \end{array}$$

96096

Vertically and crosswise

or

All from 9 and the last from 10

Proportionately

15.

$$\begin{array}{r} 2\ 7\ 3 \\ \times\ 5\ 1\ 6 \\ \hline \end{array}$$

$14_4 0_3 8_4 6_1 8$

Vertically and crosswise

16. 9.8×0.00093

$$\begin{array}{r} 9\ 8-0\ 2 \\ \times\ 9\ 3-0\ 7 \\ \hline 9\ 1\ \ 1\ 4 \end{array}$$

0.009114

All from 9 and the last from 10

17. How many pieces of wire, each 31 cm long can be cut from a roll of length 100 metres and what will be the remainder?

$3^1 \mid 10_1 0_1 0 / {}_2 0$

3 2 2/18

Vertically and crosswise

18. 16.8% of 25

$16.8\% \times 25 = 25\% \times 16.8 = 4.2$

Proportionately

19. Convert $\frac{116}{125}$ to decimal.

$$\frac{116}{125} \times \frac{8}{8} = \frac{928}{1000} = 0.928$$

20. Convert the fraction, $\frac{7}{19}$, to decimal.

$0._1 \dot{3}_1 6\,8\,4\,2\,1_1 0\,5_1 2\,6\,3\,1\,5\,7\,8\,9\,4\,\dot{7}$

By one more than the one before

21. How many different digits are there when $\frac{17}{11}$ is converted to a decimal? (Draw a ring round the correct answer.)

A 2 B 3 C 4 D 5 E 6

$\frac{17}{11} = 1\frac{6}{11} = 1.\dot{5}\dot{4}$

Transpose and apply

22.

$$\frac{1}{113} \quad \frac{2}{225} \quad \frac{4}{447} \quad \frac{2}{227}$$

$$\frac{4}{452} \quad \frac{4}{450} \quad \frac{4}{447} \quad \frac{4}{454}$$

$$\frac{2}{227} \quad \frac{1}{113} \quad \frac{2}{225} \quad \frac{4}{447}$$

Proportionately

23. Express 85 as the difference of two square numbers that are integers, in two different ways.

$85 = 5 \times 17$

$= (11-6)(11+6) = 11^2 - 6^2$

or $85 = 1 \times 85$

$= (43-42)(43+42) = 43^2 - 42^2$

By addition and subtraction

24. Tara attempted to multiply 67×58 but wrote down one digit in the two numbers incorrectly and got the answer 3596. Find the sum of the two numbers she multiplied.

$58 \mid 35_5 9_1 6$

$6\ 2/0$

$62 + 58 = 120$

Vertically and crosswise

25. Marie goes to a shop where all the items have prices ending in 99 cents (for example, \$0.99, \$5.99, \$2.99). She spends \$33.89. How many items does she buy?

11

By the deficiency

26. How many non-recurring decimal digits are there in the decimal equivalent of $\frac{101}{475}$?

$475 = 5 \times 5 \times 19$

Two non-recurring digits

All the multipliers

27. Find the cube root of the exact cube, 238,328

62

By the last digits

28. 0.79^3

$79^3 = 8\bar{1}^3$

$5\ 1\ 2 \quad \bar{6}\ \bar{4} \quad 8 \quad \bar{1}$

$\bar{1}\ \bar{2}\ \bar{8} \quad 16$

$\bar{1}\ \bar{9} \quad 2$

$5\ 0\ \bar{7} \quad 0 \quad 4 \quad \bar{1}$

$4\ 9\ 3\ 0\ 3\ 9$

Proportionately

29. Exactly one of these equations is correct. Draw a circle round the correct one.

A $44^2 + 77^2 = 4477$

B $55^2 + 66^2 = 5566$

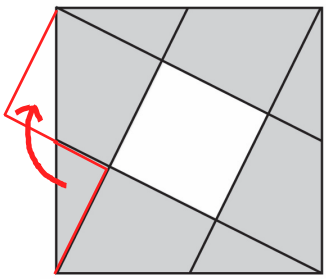
C $66^2 + 55^2 = 6655$

D $88^2 + 33^2 = 8833$

E $99^2 + 22^2 = 9922$

Only the last digits

30. What fraction of the square is left unshaded?

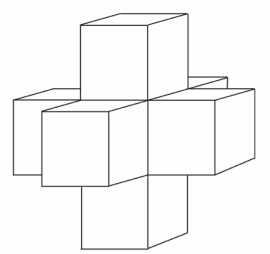


$\frac{1}{5}$ By inspection

Transpose and apply

31. The solid as shown is made by fixing cubes on each face of a central cube. The solid has a volume of $875\,cm^3$.

What is the surface area of the solid?



Volume of one cube is $875 \div 7 = 125$ cm^3

Edge length of each $= \sqrt[3]{125} = 5$ cm

Total surface area $= 5 \times 25 \times 6 = 750$ cm^2

By inspection Transpose and apply

32. Two similar cones, A and B, have surface areas $900\,cm^2$ and $8100\,cm^2$, respectively.

If the volume of cone A is $1800\,cm^3$, what is the volume of cone B?

ASF $= 1:9$, LSF $= 1:3$, VSF $= 1:27$

$27 \times 1800 = 486{,}000$ cm^3

Proportionately

33. $99^3 + 3 \times 99^2 + 9 \times 33 + 1$

$99^3 + 3 \times 99^2 + 3 \times 99 + 1$

$= (99 + 1)^3 = 1{,}000{,}000$

By inspection

34. Is 232808 divisible by 49?

2 3 2 8 0 8

97 68 62 208 40

48 19 13 12

No

By Osculation

35. If $6x - y = 21$ and $6y - x = 14$, what is the value of $x - y$?

$$7x - 7y = 7$$
$$x - y = 1$$

By addition and subtraction

36. Expand and simplify,

$$(2x^2 - 3x + 7)(3x^2 + 5x - 3)$$

$$\begin{array}{r} 2x^2 - 3x + 7 \\ \times\ 3x^2 + 5x - 3 \\ \hline 6x^4 + x^3 + 0x^2 + 26x - 21 \end{array}$$

Vertically and crosswise

37.

$$\begin{array}{r|l} x - 2 & 6x^3 - 19x^2 + 9x + 10 \\ +2 & \quad 12x^2 - 14x - 10 \\ \hline & 6x^2 - 7x - 5 \ / \ 0 \end{array}$$

$$\begin{pmatrix} 3 & -5 \\ 2 & 1 \end{pmatrix} \quad (3x-5)(2x+1)(x-2) = 0$$

$x = \frac{5}{3}, -\frac{1}{2}$ or 2

Transpose and apply

38.

$$4x^3 + 8x^2 + 9x + 10 \div (2x + 3)$$

$$\begin{array}{r|l} 2x + 3 & 4x^3 + 8x^2 + 9x + 10 \\ -3 & \quad -6x^2 - 3x - 9 \\ \hline & 2x^2 + x + 3 \ / \ 1 \end{array}$$

Transpose and apply

39. Find the equation of the straight line with gradient, 3, and that passes through the point (4, 2).

$$mx - y = mx_1 - y_1$$
$$3x - y = 10$$

Specific and general

40. Find the equation of the straight line perpendicular to the line with equation, $3x + 5y = 23$ and which passes through the point (1, 5).

$$5x - 3y = -10$$

Transpose and apply

Specific and general

41.

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \cdots = \frac{\pi}{4}$$
$$\frac{1}{1\times 3} + \frac{1}{5\times 7} + \frac{1}{9\times 11} + \cdots = \frac{\pi}{x}$$

$$\frac{1}{1\times 3} + \frac{1}{5\times 7} + \frac{1}{9\times 11} + \cdots$$
$$= \frac{1}{3} + \frac{1}{35} + \frac{1}{99} + \cdots$$
$$1 - \frac{1}{3} = \frac{2}{3}, \quad \frac{1}{5} - \frac{1}{7} = \frac{2}{35}, \quad \frac{1}{9} - \frac{1}{11} = \frac{2}{99}$$
$$x = 8$$

By inspection

42. a and b are real numbers such that,

$$a \div b = a \times b = a + b$$

Find the value of $a - b$.

$$\frac{a}{b} = ab \Rightarrow b^2 = 1 \Rightarrow b = \pm 1$$
If $b = +1$, $ab \neq a + b$
$b = -1 \Rightarrow -a = a - 1$
$a = \frac{1}{2}$ $\qquad a - b = \frac{3}{2}$

Transpose and apply

When the total is the same, there is nothing

43. Five numbers are arranged in order from least to greatest.

$$x \quad x^3 \quad x^4 \quad x^2 \quad x^0$$

Where does $-x^{-1}$ belong in the list above?

Let $x = -\frac{1}{2}$

$-\frac{1}{2} \quad -\frac{1}{8} \quad \frac{1}{16} \quad \frac{1}{4} \quad 1$

$-\dfrac{1}{-\frac{1}{2}} = 2,$ belongs at the end

Transpose and apply

44. A circle has equation, $x^2 + y^2 = 9$. In how many places does the straight line, with equation $x + 2y = 7$, cross the circumference of the circle, twice, once or no times?

$x = 7 - 2y, \quad x^2 = 49 - 28y + 4y^2$

$49 - 28y + 4y^2 + y^2 = 9$

$5y^2 - 28y + 40 = 0$

$b^2 - 4ac = 784 - 800 < 0$

No times

Transpose and apply

45. $\left(3^{23} - 3^{22}\right)\left(3^{24} - 3^{23}\right)\left(3^{25} - 3^{24}\right) = 2^m \times 3^n$

Find the value of $m + n$

$\left(3 \times 3^{22} - 3^{22}\right)\left(3 \times 3^{23} - 3^{23}\right)\left(3 \times 3^{24} - 3^{24}\right)$

$3^{22}(3-1) \times 3^{23}(3-1) \times 3^{24}(3-1)$

$= 3^{69} \times 2^3$

$m + n = 72$

By inspection

46. If a, b, c and d are the roots of the equation,

$$\left(\frac{13x - x^2}{x+1}\right)\left(x + \frac{13-x}{x+1}\right) = 42$$

Find the value of $2020(a+b+c+d)$.

$\left(\dfrac{x(13-x)}{x+1}\right)\left(x + \dfrac{(13-x)}{x+1}\right) = 42$

$\left(\dfrac{x^2(13-x)}{x+1}\right) + \left(\dfrac{x(13-x)^2}{(x+1)^2}\right) = 42$

$x^2(13-x)(x+1) + x(13-x)^2 = 42(x+1)^2$

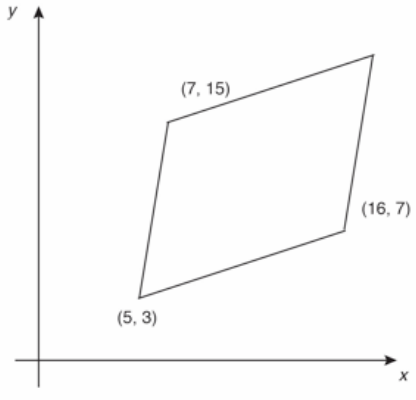
$x^4 - 13x^3 + \cdots$

$a + b + c + d = 13$

$2020(a+b+c+d) = 26260$

Transpose and apply

47. Find the area of the parallelogram given the coordinates of three vertices as shown.



$$\begin{array}{rr} (7,15) & (16,7) \\ -\ (5,3) & (5,\ 3) \\ \hline (2,12) & (11,4) \end{array}$$

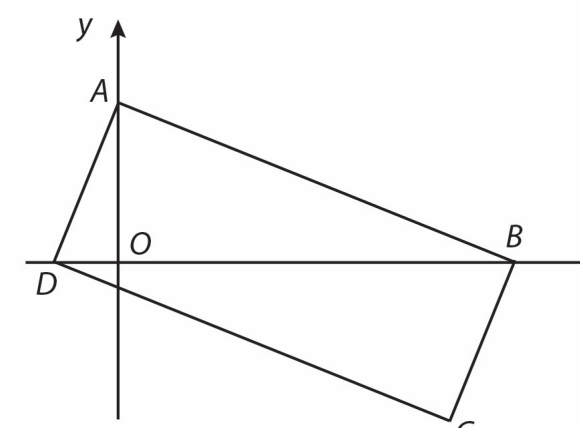
$= 132 - 8 = 124$

Transpose and apply

Product of the means
minus product of the extremes

48. The figure shows a rectangle, $ABCD$. The equation of the line AB is $2x+5y=10$. The point A lies on the y-axis and points B and D lie on the x-axis.

Work out the area of the rectangle.



A lies at $(0, 2)$, B lies at $(5, 0)$

Transpose and adjust

DA is $5x-2y=-4$, D lies at $(\frac{-4}{5}, 0)$

By elimination and retention

$(\frac{-4}{5}, 0)$ $(5, 0)$

$-$ $(0, 2)$ $(0,2)$

Product of the means
minus product of the extremes

$(\frac{-4}{5}, -2)$ $(5, -2)$

Area $= \left|-10-\frac{8}{5}\right| = 11\frac{3}{5}$

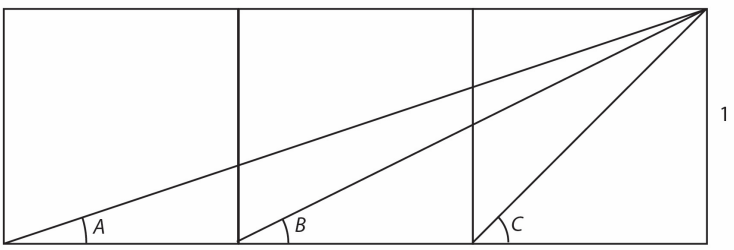
49. Evaluate,

$$\tan^{-1}\tfrac{1}{2} + \tan^{-1}\tfrac{1}{5} + \tan^{-1}\tfrac{1}{8}$$

$$\begin{array}{rr} 2 & 1 \\ +\ 5 & 1 \\ \hline 9 & 7 \\ +\ 8 & 1 \\ \hline 65 & 65 \end{array} = 45^\circ$$

Vertically and crosswise

50. Find the sum of the three angles A, B and C.



$$\begin{array}{rr} 3 & 1 \\ +\ 2 & 1 \\ \hline 5 & 5 \\ +\ 1 & 1 \\ \hline 0 & 10 \end{array}$$

90°

Vertically and crosswise

Topics

Subtraction	Divisibility by 9
Nikhilam multiplication, below the base, above the base, above and below the base, working bases, and application to decimal products	Multiplication of two-digit numbers when the final digits add to 10
Squaring numbers close to a power of 10.	Squaring two-digit numbers ending in 5
Multiplying and dividing by 5	Vertically and crosswise multiplication up to three-digit numbers
Nikhilam division	Paravartya division, numerical and algebraic
Straight division	Conversion of decimals to fractions
Conversion of fractions to decimals using Proportionately and By one more than the one before.	Comparing fractions
Express a number as the difference of two squares	Using deficiencoes to solve problems
The factors determining non-recurring digits in partly-recurring decimals	Cube roots of exact cubes of two-digit numbers
Cubing two-digit numbers	Using final digits to check calculations
Transposing shapes to solve geometric problems	Using linear scale fact, area scale factor and volume scale factor to solve problems involving similar volumes
Cubes of binomials	Using osculation to test for divisibility
Simultaneous equations	Multiplying trinomials by Vertically and crosswise
Solving cubing equations	Equation of straight line given one point and gradient
Equation of straight line perpendicular to a given line and given one point	Problems involving infinite series
Equation of a circle. Use of discriminant to solve geometric problems	Problems involving laws of indices
Sum of roots of polynomials	Area of parallelogram, given coordinates of vertices
Addition of Triples to determine angles	