Some Aspects Of Surds

After a period of research I have found new methods to solve different problems in higher mathematics using the sutras given by Jagadguruji. This article shows a few examples in the topic of surds.

Example: Which is bigger $\sqrt{22}+\sqrt{21}$ or $\sqrt{23}+\sqrt{20}$?

The question is dependent upon the individual numbers on either side having the same sum

Conventional method:

$$\sqrt{22}+\sqrt{21} >=< \sqrt{23}+\sqrt{20}$$
$$22+21+2\sqrt{462} >=< 23+20+2\sqrt{460}$$
$$43+2\sqrt{462} >=< 43+2\sqrt{460}$$
$$2\sqrt{462} >=< 2\sqrt{460}$$
$$\sqrt{462} > \sqrt{460}$$
$$\therefore \sqrt{22}+\sqrt{21} > \sqrt{23}+\sqrt{20}$$

Vedic method:

In the Vedic method we use the sutra संकलन व्यवकलनाभ्याम् - Sankalana Vyavakalanabhyam – *By addition and subtraction*.

To solve $\sqrt{22}+\sqrt{21} >=< \sqrt{23}+\sqrt{20}$ first we add both side numbers without taking the square root.

Addition	43	43
Subtraction	1	3
Difference	42	40

As 42 > 40, so $\sqrt{22}+\sqrt{21} > \sqrt{23}+\sqrt{20}$.

Below we give two more examples:

$\sqrt{17}+\sqrt{20} >=< \sqrt{7}+\sqrt{30}$

By addition	37	37
By subtraction	3	23
Difference	34	14

$34 > 14$

So $\sqrt{17}+\sqrt{20} > \sqrt{7}+\sqrt{30}$

$\sqrt{13}-\sqrt{11} >=< \sqrt{7}-\sqrt{5}$

or $\sqrt{13}+\sqrt{5} >=< \sqrt{7}+\sqrt{11}$

By addition	18	18
By subtraction	8	4
Difference	10	14

$10 < 14$

So $\sqrt{13}-\sqrt{11} < \sqrt{7}-\sqrt{5}$

Computation of square root (two values)

Example: Simplify $\sqrt{24+8\sqrt{3}}$.

Conventional method:

$$\text{Let} \quad \sqrt{14+8\sqrt{3}} = \sqrt{x} + \sqrt{y}$$

$$\text{Square both sides,} \quad 14+8\sqrt{3} = x + y + 2\sqrt{xy}$$

$$\text{or} \quad 14+2\sqrt{48} = x + y + 2\sqrt{xy}$$

$$\text{So} \quad x + y = 14 \quad \text{and} \quad xy = 48$$

$$\text{Now since} \quad (x-y)^2 = (x+y)^2 - 4xy$$

$$\text{then} \quad x - y = \sqrt{(x+y)^2 - 4xy}$$

$$= \sqrt{14^2 - 4\times 48} = \sqrt{4} = 2$$

$$\text{As} \quad x + y = 14 \ \text{and} \ x - y = 2$$

$$\text{then} \quad x = \frac{14+2}{2} = 8 \ \text{and} \ y = \frac{14-2}{2} = 6$$

$$\text{So} \ \sqrt{14+8\sqrt{3}} = \sqrt{8} + \sqrt{6} = 2\sqrt{2} + \sqrt{6}$$

Vedic method:
Using the same *By addition and subtraction* sutra as before,

$$\text{Let} \quad \sqrt{14+8\sqrt{3}} = \sqrt{a} + \sqrt{b}$$

$$\text{Let} \quad x = 14 \quad \text{and} \quad y = \sqrt{14^2 - 192} = 2$$

$$\text{Now} \quad a = \frac{x+y}{2} = \frac{14+2}{2} = 8$$

$$\text{and} \quad b = \frac{x-y}{2} = \frac{14-2}{2} = 6$$

$$\therefore \ \sqrt{14+8\sqrt{3}} = \sqrt{8} + \sqrt{6} = 2\sqrt{2} + \sqrt{6}$$

Example: Simplify $\sqrt{11-4\sqrt{7}}$

$$\sqrt{11-4\sqrt{7}} = \sqrt{11-\sqrt{112}}$$

$$x = 11, \quad y = \sqrt{11^2 - 112} = 3$$

$$a = \frac{11+3}{2} = 7, \quad b = \frac{11-3}{2} = 4$$

$$\therefore \sqrt{11-4\sqrt{7}} = \sqrt{7} - \sqrt{4} = \sqrt{7} - 2$$

Example:

$$\sqrt{2x^2 - y^2 + 2x\sqrt{x^2 - y^2}} \quad = \quad \sqrt{(2x^2 - y^2) + \sqrt{4x^4 - 4x^2y^2}}$$

$$X = 2x^2 - y^2, \qquad Y \quad = \quad \sqrt{(2x^2 - y^2)^2 - (4x^4 - 4x^2y^2)}$$

$$= \quad \sqrt{4x^4 - 4x^2y^2 + y^4 - 4x^4 + 4x^2y^2} \quad = \quad \sqrt{y^4} \quad = \quad y^2$$

$$a = \frac{(2x^2 - y^2) + y^2}{2} = x^2, \qquad b = \frac{(2x^2 - y^2) - y^2}{2} = x^2 - y^2$$

$$\therefore \sqrt{2x^2 - y^2 + 2x\sqrt{x^2 - y^2}} \quad = \quad \sqrt{x^2} + \sqrt{x^2 - y^2} \quad = \quad x + \sqrt{x^2 - y^2}$$

Example: $\sqrt[4]{17+12\sqrt{2}}$

$$\sqrt[4]{17+12\sqrt{2}} = (17+\sqrt{288})^{\frac{1}{4}}$$

$$X = 17, \quad Y = \sqrt{17^2 - 288} = 1$$

$$a = \frac{17+1}{2} = 9, \quad b = \frac{17-1}{2} = 8$$

$$\therefore (17+\sqrt{288})^{\frac{1}{4}} \quad = \quad (\sqrt{9}+\sqrt{8})^{\frac{1}{2}} \quad = \quad \sqrt{3+\sqrt{8}}$$

$$X = 3, \quad Y = \sqrt{3^2 - 8} = 1$$

$$a = \frac{3+1}{2} = 2, \quad b = \frac{3-1}{2} = 1$$

$$\therefore \sqrt{3+\sqrt{8}} \quad = \quad \sqrt{2}+\sqrt{1} \quad = \quad 1+\sqrt{2}$$

$$\text{So } \sqrt[4]{17+12\sqrt{2}} \quad = \quad 1+\sqrt{2}$$

Further aspects of surds and higher mathematics together with proofs of these methods will be supplied in the next newsletter.

Panna Patodia, January 2004

Square root of complex numbers

The Vedic method applied to surds by Mr. Panna Patodia (Newsletter No.12) can be applied to complex numbers also using the same logic.

Suppose we have to find $\sqrt{-9+40i}$.

$$\text{Let } \sqrt{-9+40i} = a+bi$$

$$\text{Let } x = -9 \text{ and } y = \sqrt{(-9)^2 - (40i)^2} = \sqrt{1681} = 41$$

$$\text{then } a = \sqrt{\frac{x+y}{2}} = \sqrt{\frac{-9+41}{2}} = \pm 4$$

$$\text{and } b = \sqrt{\frac{y-x}{2}} = \sqrt{\frac{41-(-9)}{2}} = \pm 5$$

$$\therefore \sqrt{-9+40i} = \pm(4+5i)$$

Thanks Mr. Panna Patodia for the beautiful method. This would definitely be appreciated by higher secondary students .