Vedic Maths as a Pedagogic Tool

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Abstract

The characteristics of Vedic mathematics lead to a teaching and learning approach that has many advantages over conventional pedagogies. These include speed, creativity, flexibility and the development of strategy skills in problem solving.

Due to exam syllabus requirements students often merely follow rules for solving problems at the expense of understanding. However, the Vedic approach enables students, not only to excel in their learning and exam performance, but also to enjoy the subject and to explore and apply their own creativity. The sutra-based system contains simple yet subtle principles each with multifarious applications. This enables pattern-recognition to develop across different topics.

One of the most sort after uses of mathematics education is the training it can provide in managing problems using alternative strategies. This is one of the hallmarks of Vedic mathematics, for not only does it have over-arching methods, but also special case methods which are brought to bear when dealing with particular problems.

This paper provides examples showing the properties and qualities of Vedic Maths and discusses why it is a valuable pedagogic tool.

Introduction

The Vedic system has several facets, that are explored in this paper, that render it ideal not only for improving understanding mathematics but also for teaching and learning the subject in a more holistic, natural and practical way. The variety of techniques offered leads to creativity, increased speed of mental work and a better understanding of the subject. These also empower students more which leads to a different attitude to their approach to mathematics and therefore greater appreciation and enjoyment of the subject.

Speed

The mind works extremely fast so that in written work there is a discrepancy between the rate at which mental impressions are experienced and the rate at which these impressions can be recorded on paper. This leads to a dichotomy which does not exist when calculations are performed mentally. Since the Vedic system works the way the mind works it is necessarily the most efficient system for mental calculations. There are therefore potentially huge benefits in developing quick thinking, mental agility, memory etc. through use of the Vedic system.

Moreover, even in written work where the mind needs to coordinate with the body as well as translate into meaningful symbols, the simple patterns and direct thinking mean more mental energy is available to deal with these extra challenges.

Anecdotally, in our experience of teaching Vedic techniques to students they are often very surprised and delighted with the speed at which they can complete calculations, particularly where they are trained and well-versed in conventional methods.

As an example of how fast the Vedic techniques can be, consider multiplying 888 by 997. The conventional method is long multiplication as shown below.

$$\begin{array}{r} 8\,8\,8 \\ \times\ 9\,9\,7 \\ \hline 6\ 2_6 1_5 6 \\ \hline 7\ 9_7 9_7 2\ 0 \\ \hline 7\,9_7 9_7 2\ 0\ 0 \\ \hline 8\ 8\ 5\ 3\ 3\ 6 \end{array}$$

The Vedic one-line method looks like this:

$$\begin{array}{r} 8\,8\,8-1\,1\,2 \\ \times\ 9\,9\,7-0\,0\,3 \\ \hline 8\,8\,5\quad 3\,3\,6 \end{array}$$

Another striking example can be seen with the following problem in coordinate geometry. The diagram shows a parallelogram with three vertices at (2, 3), (5, 9) and (7, 4). The problem is to find the area.

This uses two Vedic Maths sutras, *Transpose and adjust* and *The product of the means minus the product of the extremes*[1]. The application of these produces a neat solution that can be calculated mentally.

The first step is to transpose one vertex to the origin, say (2, 3) with the consequence that two other vertices are repositioned. This requires subtracting the x-coordinate, 2, and the y-coordinate, 3, from the coordinates of the other two vertices. This results in (3, 6) and (5, 1).

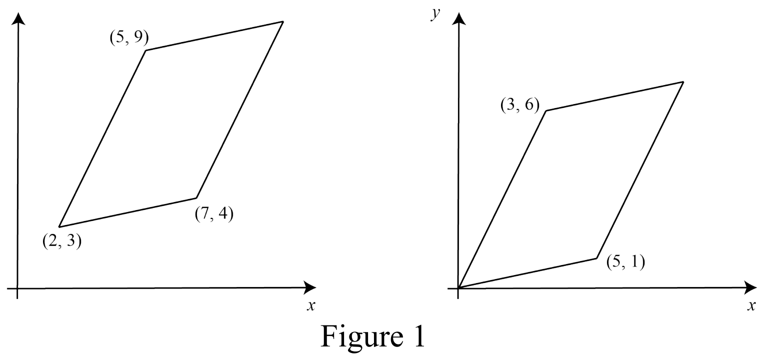

Figure 1

$$\begin{array}{c} (3,\ 6)\quad (5,\ 1) \\ 30-3=27 \end{array}$$

Applying the second sutra as $6\times5-3\times1=27$ yields the answer.

Flexibility
In a fast-changing world, where technology updates rapidly and a ‘job for life’ is no longer an option, adaptability and flexibility are essential requirements for success. The Vedic system teaches this through multiple ways of looking at a problem, multiple ways to arrive at an answer and even multiple ways to express numbers. We can work two or more figures at a time, work left to right of right to left, or in some cases from the middle outwards. Far from confusing the student, this leads to them being more engaged in their study because they know from early in their school career that the work is easier the more tools they have. And if they prefer to exercise very few tools that is fine too, but they may have to work a little harder.

A simple example of the flexibility of the system can be seen in eight methods for squaring 57.

Method A Using 50 as a working base and 100 is the real base.
$57^2 = 64(\div 2)/49 = 3249$
The surplus of 7 is added to 57 giving 64. The square of 7 is 49.
The relationship of the working base to the real base requires division of the left-hand part by 2, giving 3249.

Method B Using 50 as a working base and 10 as the real base
$57^2 = 64(\times 5)/{}_49 = 320/{}_49 = 3249$
Again the surplus of 7 is added giving 64 on the left. Since 10 is the real base, the square of 7 is only allowed one digit’s space and so 49 is written as 9 with 4 to carry.
The relationship of 50 to the real base requires multiplication of the left-hand part by 5 giving 320. 4 is carried leaving 3249.

Method C Using 60 as the working base and 10 as the real base
$57^2 = 54(\times 6)/9 = 3249$
The deficiency of 57 from 60 is 3 and 57 is then decreased by this amount giving 54. The square of the deficiency is 9, which gives the right-hand part.
The left-hand part is then multiplied by 6 to give 324.

Method D Using Straight Squaring
$57^2 = 25/{}_70/{}_49 = 3249$
This is a condensed form of multiplying by Vertically and Crosswise and the answer comes in three parts. The square of 5 is 25. The middle part is found by multiplying 5 by 7 and doubling the answer, 70, which is 0 with 7 to carry. The right-hand part is the square of 7, put down as 9 with 4 to carry. Once the carry digits are absorbed the answer is 3249.

Method E Straight Squaring using a vinculum
$6\bar{3}^2 = 36/{}_{\bar{3}}\bar{6}/9 = 33\bar{6}9 = 3249$
57 is 3 less than 60 and, using a vinculum (a minus), can be written as $6\bar{3}$. The straight squaring method is then applied.

Behind each of these methods lies an algebraic explanation that draws the student towards a deeper understanding not only of the relationship between number and algebra but also the principles and use of algebra. For example, the algebra behind the method of straight squaring is that $(a+b)^2 = a^2 + 2ab + b^2$.

Method F Using the Proportionately rule

$$\begin{array}{rcccc} 57^2 & = & 25 & 35 & 49 \\ & & & 35 & \\ \hline & & 3\ 2 & {}_7 4 & {}_4 9 \end{array}$$

The square of the first digit is set down followed by two successive terms of the geometric sequence using the ratio of the two digits, 5 : 7. The middle term is doubled and then the digits are summed.

Method G Proportionately with vinculum

$$\begin{array}{rcccc} 6\bar{3}^2 & = & 36 & \bar{1}\bar{8} & 9 \\ & & & \bar{1}\bar{8} & \\ \hline & & 36 & {}_3\bar{6} & 9 \\ \hline & & 32 & 4 & 9 \end{array}$$

This is the same as the previous method but using a vinculum digit.

Method H Difference of two squares

$$\begin{aligned} 57^2 - 3^2 &= (57-3)(57+3) \\ 57^2 &= 54 \times 60 + 9 \\ &= 3240 + 9 \\ &= 3249 \end{aligned}$$

By choosing a suitable partner, 3 in this case, the square is found through an easy product and addition. The sutra in this case is *By addition and subtraction*.

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Strategy skills
The Vedic Sutras describe the natural ways in which the mind works: we might say they describe the natural strategies brought to bear when tackling problems. The five methods above illustrate how a simple problem can be tackled in a number of ways. By developing these skills (of creating multiple possible strategies) each student gains the freedom of choosing whichever method they like. The English essayist Joseph Addison wrote, “Nature delights in the most plain and simple diet”.[2]
He was expressing the re-emerging idea of the principle of least action which, although of Greek antiquity origin, had been taken up in the 17th century by both Newton and Fermat.

It appears that, built into nature, there is an instinctive tendency to follow the path of least action and this will most naturally decide whichever method is personally felt to be the easiest.

Moreover, a single example like this illustrates how there are often different strategies for solving a problem, and students then begin to look for various strategies for solving other problems.

Referring back to the previous problem of finding the area of a parallelogram with given coordinates, when first presented, students invariably try to use their knowledge that the area of a parallelogram is base times height and then start trying to work out those lengths. On reaching failure the next strategy is to find the area of the containing rectangle and then subtract four trapezia and the small rectangle as shown in Figure 2. The whole is the sum of the parts , which is one aspect of the *Specific and General* sutra. This works but is a cumbersome affair by comparison with the previous method.

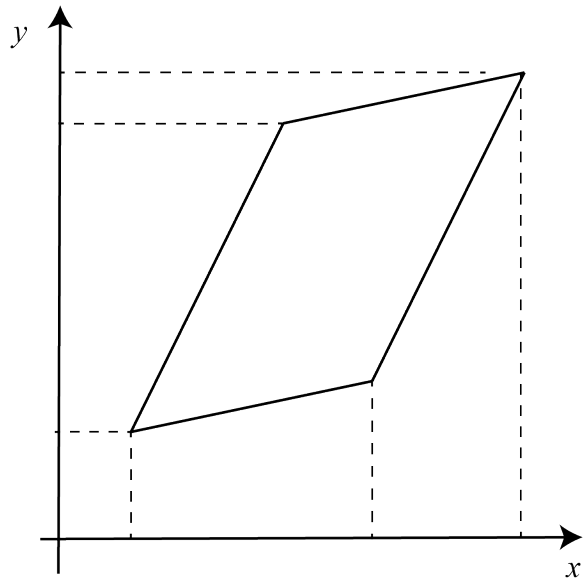

Figure 2

The importance of developing strategy skills in mathematics education cannot be overemphasised and such skills are a hallmark of the Vedic system. By practising and exercising these Sutras we enliven those natural faculties so that they can be used in all activities we engage in. In this way the Vedic system is ideal for developing problem solving skills.

The need to anticipate and appreciate the effects that one's actions will have, or may have, in the future is also an essential element in education. The Vedic system is ideal for teaching this skill: through its multiple approaches students appreciate the advantages of looking ahead in order to be most effective. For example, in subtracting 191 from 323 we may think at first that the first figure will be '2', but looking ahead at the next column we see that it cannot be '2' and that '1' is a better choice.

Creativity

The use of special methods in the Vedic system leads naturally to opportunities to have a personal influence in dealing with mathematics. Students like to have their own input as they are naturally creative. This makes the study of mathematics more fulfilling and also leads to a better understanding of the laws of mathematics because students are more involved and attentive. Those of a more artistic temperament, who normally find themselves not attracted to mathematics work find it appeals to them because of this creative dimension. It also encourages the use of intuition, which is normally discouraged in the maths class.

An example of this is found in the following problem: What is the minimum number of cuts required to divide a cube into 27 equal cubes?

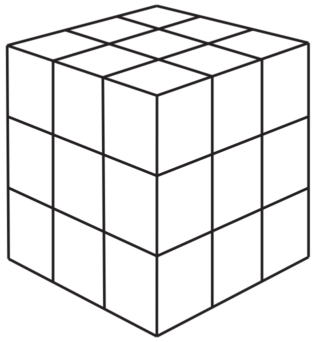

Figure 3

A possible solution might arise from considering slicing one third of the cube and then placing the separated part on top in some way before making another cut in the attempt to minimise the number. But the problem is easily solved with a more intuitive and creative approach which is to consider the small cube at the centre. It has six faces and so the minimum number of cuts is six.

Improving understanding

With the Vedic Sutras, the mental approach, and the flexibility which the Vedic system affords, understanding of mathematics is deeper and more connected. The Sutras lead to comprehension, and thinking that is more natural and unified. In calculating mentally, which is a natural preference in the Vedic system, the subtle properties of numbers, etc., are appreciated much more readily than if a calculation was written down and thereby fixed. And the multiplicity of approaches to any topic means students see them from different angles, so that different properties and aspects are appreciated and highlighted, which inevitably increases understanding.

In addition to this, the fast methods, which at first seem magical, beckon the students to understand the mathematics. The example cited above, of finding the area of a parallelogram, has a very simple and surprising solution and yet behind it lies a beautiful element of mathematics related to determinants of matrices and vector products.

One of the key aspects is the connection between algebra and arithmetic. Each of the Vedic methods has an algebraic counterpart. For example, the method for algebraic division uses the *Transpose and Adjust* sutra and is, in fact, a condensed form of conventional division. This method is directly applied to number by letting the variable x be equal to 10.

For example, $x^3 + 5x^2 + 8x + 9$ can be divided by $x + 2$ to give $x^2 + 3x + 2$ remainder 5. The same method can also be applied to $1589 \div 12 = 132 / 5$. In this way the understanding of the arithmetic supports that of the algebra and vice versa. The algebra is appreciated as dealing with a more generalised form and arithmetic with the particular.

In developed countries calculators are widely used in many schools. Although they have their place in facilitating easy access to various calculations such as with trigonometry, they do little to advance understanding of mathematics. The methods within Vedic maths, on the other hand, lead to a deeper penetration of underlying concepts. Furthermore, calculators and computers are only accessible to a fraction of the estimated 2 billion school-aged children in the world, all of whom can benefit from learning mathematics.

Pattern recognition within different topics

Much of mathematics concerns finding patterns. In his book *Prelude to Mathematics*, W. W. Sawyer states, "...in nature we sometimes find the same pattern again and again in different contexts, as if the supply of suitable patterns were extremely limited"[3]. A similar view was expressed by Poincare when he said, "Mathematics is the art of giving the same name to different things". To investigate mathematics in this way is a move towards unity, away from diversity. The sutras of Vedic mathematics powerfully demonstrate the same principle. They are frequently seen to apply in diverse contexts which then become unified. So a single sutra can describe a pattern of mental process, or shape of a formula, which is then repeated in disconnected topics. The topics then become unified, not because the mathematics are logically connected but because they are *intuitively* connected.

Several examples of this are seen with the *Vertically and Crosswise* sutra.

Initially, it is used for multiplying numbers or polynomials of any size in one line.

$$\begin{array}{r} 3\,4\,2 \\ \times\ 6\,2\,7 \\ \hline 2\,1_3 4_4 4_3 3\,_1 4 \end{array} \qquad\qquad \begin{array}{r} 2x^2-3x+5 \\ \times\ 3x^2+2x-1 \\ \hline 6x^4-5x^3+7x^2+13x-5 \end{array}$$

But it also has many other applications in diverse topics. For example, the same pattern is used for combining fractions:

$$\frac{3}{4}+\frac{2}{5} = \frac{15+8}{20} \qquad\qquad \frac{2x}{3y}+\frac{5y}{4x} = \frac{8x^2+15y^2}{12xy}$$

Another example occurs in differentiating a product:

$$\begin{array}{c} (4+\sin x)e^{2x} \\ \cos x \quad 2e^{2x} \\ \hline 2e^{2x}(4+\sin x)+e^{2x}\cos x \end{array}$$

In this case the derivatives are written underneath and the answer is the sum of the cross-products.

Structures that link different areas abound in mathematics and need to be recognized and utilized. They bring out the beauty and unity in the subject and can also aid tremendously in our understanding and knowledge.

Concluding discussion

In our experience the Vedic system appears to be effective with just about everyone: one child may love the choice and freedom to experiment while another may prefer to stick to the general methods and enjoy the simple patterns they can use. Artistic types love the opportunity to invent and have their own unique input, while the more analytic personality enjoys the challenge and scope of multiple methods.

So when we find that we can divide by simply reversing multiplication, and that the techniques we discover in arithmetic can be applied in algebra, this is very satisfying and leads to further interest as well as expanding the view of the subject as a whole.

The many advantages offered by Vedic mathematics need to be put to use in our education system as this has the potential as Swami Tirtha himself says to "turn mathematics for the children from its present excruciatingly painful character to the exhilaratingly pleasant and even funny and delightful character it really bears."[4]

The coherence and simple structures in mathematics have always been there, just as the human mind has presumably always operated in the way it does. In fact it is likely that the mind impresses its own nature onto the objects that it experiences and thereby imposes the structures that are observed in mathematics. The correlation between the Vedic Sutras and modes of mental working would then necessarily give the most efficient system possible as well as the most pleasant to use.

References

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[1] Joseph Addison, *The Spectator, Vol I, 1826, No 162, p. 255*
[2] Bharati Krishna Tirtha, Vedic Mathematics, Motilal Banarsidass, 1956, p356
[3] WW Sawyer, Prelude to Mathematics, Penguin, 1955, p13
[4] Bharati Krishna Tirtha, Vedic Mathematics, Motilal Banarsidass, 1956, p239